Tensile Failure of Fibrous Composites

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This paper presents a theoretical and experimental treatment of the failure of a composite, consisting of a matrix stiffened by uniaxially oriented fibers, when subjected to a uniaxial tensile load parallel to the fiber direction. The fibers are treated as having a statistical distribution of flaws or imperfections that result in fiber failure under applied stress. The statistical accumulation of such flaws within a composite material is demonstrated to be the cause of composite failure. An experimental program utilizing a new technique to observe the failure process is also described. The test specimens contain a single layer of glass fibers which enables microscopic evaluation of the internal failure process. Random fiber fractures are observed at loads significantly below the utimate composite strength level.

Introduction

Composite materials consisting of a ductile matrix reinforced by high-strength, high-stiffness fibers are materials of considerable engineering practicality. The strength of such materials under tensile loads has been studied theoretically with only limited success. An analytical understanding of the failure of such materials is desirable, not only to provide adequate design methods for existing materials, but also to enable the definition of desirable characteristics of constituents of composites for future applications. This paper treats the failure of a composite, consisting of a matrix stiffened by uniaxially oriented fibers when subjected to a uniaxial tensile load parallel to the fiber direction.

The failure of a uniaxially stiffened matrix has been studied previously by several investigators.[1] The simplest failure model treated assumes that a uniform strain exists throughout the composite and that fracture occurs at the failure strain of the fibers alone.[2] In this approach, no consideration is given to the fact that failure strain for a given fiber population is not a unique quantity. The effect of a fiber strength distribution was studied by Parratt,[3] who suggests the influence of fiber flaws on composite failure. In his model, failure occurs when the accumulation of fiber fractures resulting from increasing load shortens the fiber lengths to the point that further increases in load could not be transmitted to the fibers because the maximum matrix shear stress was exceeded. Thus, composite failure results when a shear failure of the matrix or the interface occurs.

In the present paper, fibers are treated as having a statistical distribution of flaws or imperfections, which result in fiber failure at various stress levels. Composite failure occurs when the remaining unbroken fibers, at the weakest cross section, are unable to resist the applied load. Thus, composite failure results from tensile fracture of the fibers. The composite strength is evaluated herein as a function of the statistical strength characteristics of the fiber population and of the significant parameters defining composite geometry. A quantitative application is presented for glass fiber reinforced plastic composites utilizing existing data for tensile strength of glass fibers. An experimental program undertaken to qualify the analysis is also described. The test specimens were observed microscopically to enable evaluation of the internal failure process.

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Description of the Model

The composite treated is shown in Fig. 1 and consists of parallel fibers in an otherwise homogeneous matrix. The fibers are treated as having a statistical distribution of flaws or imperfections that result in fiber failure under applied stress. The statistical accumulation of such flaws within a composite material results in composite failure. The computation of stress is quite complex when there are discontinuous fibers present. These internal discontinuities result in shear stresses that locally may attain very high values. An exact evaluation of this stress distribution for the complex geometry of circular cross-section fibers randomly arrayed within a matrix appears to be unattainable from a practical viewpoint. Such stresses have been evaluated for idealized fiber shape and without the effect of surrounding fibers.[4] An approximate solution, similar to that of Dow,[5] but including the effect of surrounding fibers, is obtained herein.

In the present model, the extensional stresses in the matrix are neglected relative to those in the fiber, and the shear strains in the fiber are neglected relative to those in the matrix. This approximation of the model is considered appropriate for fibers that are very strong and stiff relative to the matrix. In the vicinity of an internal fiber end, in such a composite (Fig. 1), the axial load carried by the fiber is transmitted by shear through the matrix to adjacent fibers. A portion of the fiber at each end is, therefore, not fully effective in resisting the applied stress. As the fibers are loaded, failure occurs at points of imperfection along the fibers. Increasing load produces an increasing accumulation of fiber fractures until a sufficient number of ineffective

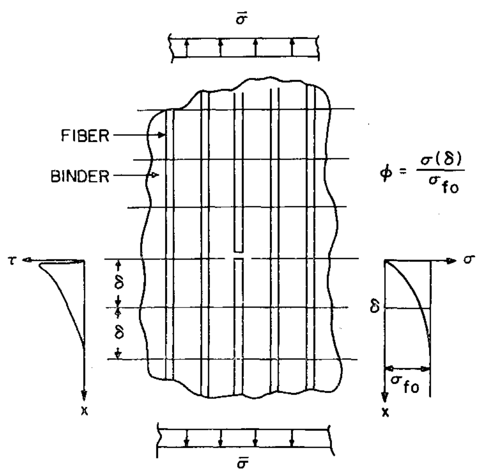

Fig. 1 Fiber reinforced composite: failure model.

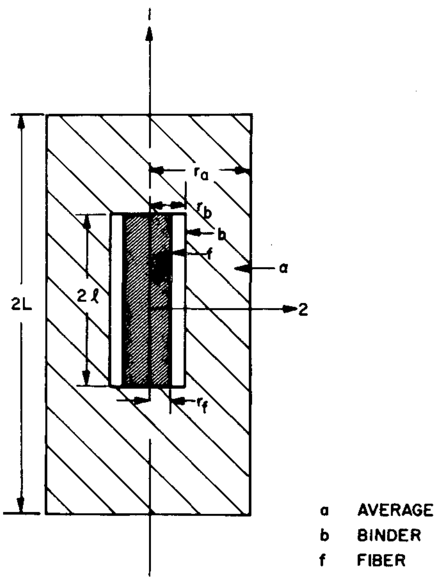

Fig. 2 Model for evaluation of stresses at fiber ends.

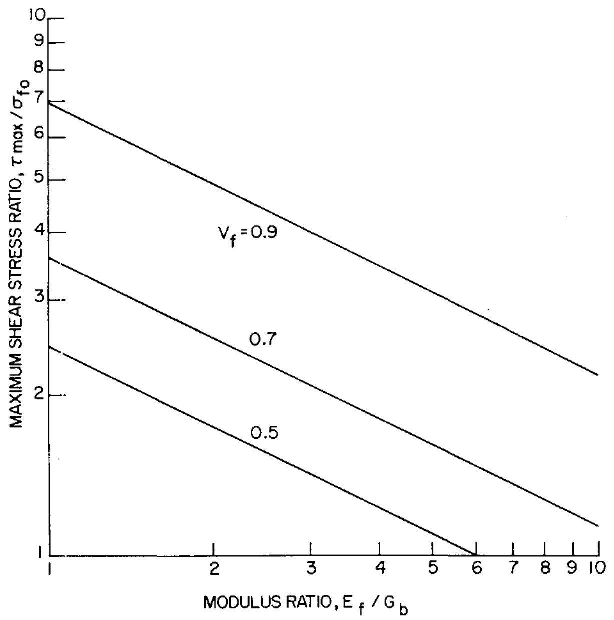

Fig. 4 Maximum fiber-matrix interface shear stress.

fiber lengths combine to produce a weak surface and composite fracture. Basically, then, the model considers fibers that fail as a result of statistically distributed flaws or imperfections and composites that fail as a result of a statistical accumulation of such flaws over a given region.

At some distance from an internal fiber break, the fiber stress will be a given fraction ϕ of the undisturbed fiber stress σ_{f0}. One may define this fraction of the average stress such that the fiber length δ over which the stress σ is less than $\phi\sigma_{f0}$ may be considered ineffective. Thus, this ineffective length δ is defined as $\sigma(\delta) = \phi\sigma_{f0}$.

Then, the composite may be considered to be composed of a series of layers of dimension δ. Any fiber that fractures within this layer, in addition to being unable to transmit a load across the layer, will also not be stressed within that layer to more than the stress $\phi\sigma_{f0}$. The applied load is treated as uniformly distributed among the unbroken fibers in each layer. The segment of a fiber within a layer may be considered as a link in the chain that constitutes the fiber.

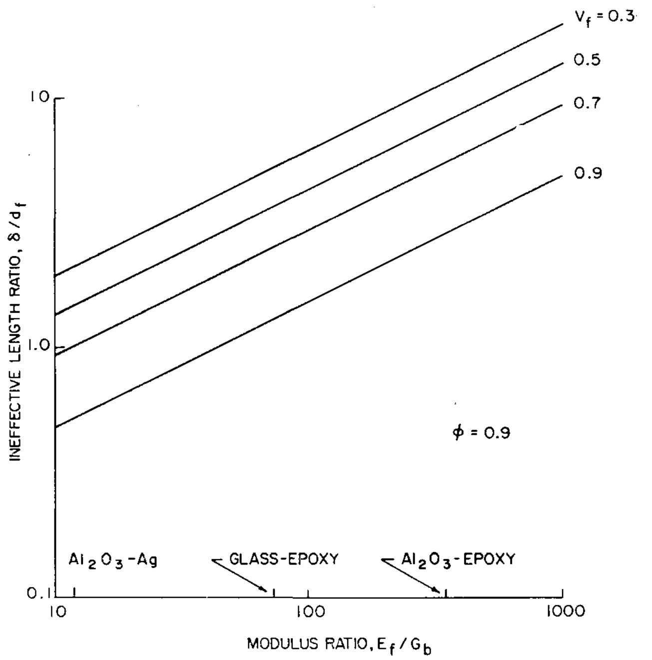

Fig. 3 Ineffective length of fibers in composites.

Each layer is then a bundle of such links, and the composite is a series of such bundles.

The treatment of a fiber as a chain of links is appropriate to the hypothesis that fracture is a result of local imperfections. The links may be considered to have a statistical strength distribution that is equivalent to the statistical flaw distribution along the fibers. The realism of such a model is demonstrated by the length dependence of fiber strength; that is, longer chains have a high probability of having a weaker link than shorter chains, and this agrees with experimental data[6] that demonstrate that fiber strength is a monotonically decreasing function of fiber length.

For this model, the link dimension is defined by a shear-lag-type approximate analysis of the stress distribution in the vicinity of a broken end. The statistical strength distribution of the links is then expressed as a function of the fiber strength-length relationship, which can be experimentally determined. Then these results are used in a statistical study of a series of bundles of links to define the distribution of bundle strengths. (Statistical techniques for a series of bundles have been studied previously for application to particle reinforced materials.[7]) The composite fails when any bundle fails, and the composite strength is thus determined as a function of fiber and matrix characteristics. These aspects of the problem are discussed in further detail below.

Statistical Analysis of the Model

The model, as described in the previous section, consists of a chain of bundles of fiber links. The links have a length δ, which is to be determined subsequently, and they are characterized by a distribution function $f(\sigma)$ and the associated cumulative distribution function $F(\sigma)$, where

$$F(\sigma) = \int_0^\sigma f(\sigma)d\sigma \qquad (1)$$

and σ is the fiber stress. The experimental method for defining the distribution function will be described subsequently. With this distribution known, the distribution function for bundle strength can be obtained; then the composite will be treated as a chain of bundles, and weakest link statistical theorems will be applied. This leads to the desired statistical definition of composite strength.

For a bundle of links, Daniels[8] has shown that, for a large number N of fibers, the distribution of the average fiber

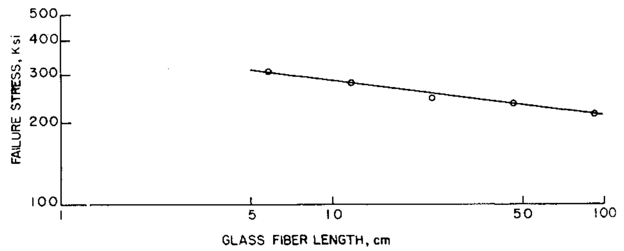

Fig. 5 Effect of length on mean fiber strength (from data of Ref. 6).

stress at bundle failure σ_B approaches a normal distribution with expectation

$$\bar{\sigma}_B = \sigma_m[1 - F(\sigma_m)] \tag{2}$$

and standard deviation

$$\psi_B = \sigma_m\{F(\sigma_m)[1 - F(\sigma_m)]\}^{1/2}N^{-1/2} \tag{3}$$

The associated density distribution function is thus

$$\omega(\sigma_B) = \frac{1}{\psi_B(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{\sigma_B - \bar{\sigma}_B}{\psi_B}\right)^2\right] \tag{4}$$

The maximum fiber stress σ_m is evaluated by maximizing the total load, which may be expressed as the product of the fiber stress and the number of unbroken fibers. Hence,

$$(d/d\sigma)\{\sigma[1 - F(\sigma)]\}_{\sigma=\sigma_m} = 0 \tag{5}$$

With the bundles characterized by Eq. (4) and the associated cumulative distribution function $\Omega(\sigma_B)$ given by

$$\Omega(\sigma_B) = \int_0^{\sigma_B} \omega(\sigma_B)\, d\sigma_B \tag{6}$$

the bundles may be treated as links in a chain, and weakest link theorems can be applied to define composite failure. For m elements forming a chain, the distribution function $\lambda(\sigma_c)$, for the average fiber stress at composite failure σ_c, is defined by

$$\lambda(\sigma_c) = m\omega(\sigma_c)[1 - \Omega(\sigma_c)]^{m-1} \tag{7}$$

That is, $\lambda(\sigma_c)d\sigma_c$ is obtained by multiplying the probability that one bundle fails between σ_c and $\sigma_c + d\sigma_c$ (which is equal to $\omega(\sigma_c)d\sigma_c$), by the probability that all remaining $(m - 1)$ elements exceed $\sigma_c + d\sigma_c$ in strength (which is equal to $[1 - \Omega(\sigma_c)]^{m-1}$), and by considering that failure can occur in any of the m bundles. This approach will be applied to glass reinforced plastic composites below as a typical example.

Fiber Link Length

The definition of ineffective length δ involves the determination of the shear stress distribution along the fiber-matrix interface. The model used is shown in Fig. 2 and consists of a fiber surrounded by a matrix that in turn is imbedded within a composite material. The latter has the average or effective properties of the composite under consideration. This configuration is subject to axial stress, and a shear-lag-type analysis is utilized to estimate the stresses. Load is applied parallel to the fiber direction. It is assumed that no stress is transmitted axially from the fiber end to the average material. Extensional stresses in the matrix material are neglected. Also, shear strains in the fiber and average materials are assumed to decay in a negligible distance from the respective interfaces with the matrix material. The shear stresses τ that result from this analysis[9] are given by

$$\frac{\tau}{\sigma_{f0}} = \frac{1}{2}\left(\frac{G_b}{E_f}\right)^{1/2}\left(\frac{v_f^{1/2}}{1 - v_f^{1/2}}\right)^{1/2}(\cosh\eta x - \sinh\eta x) \tag{8}$$

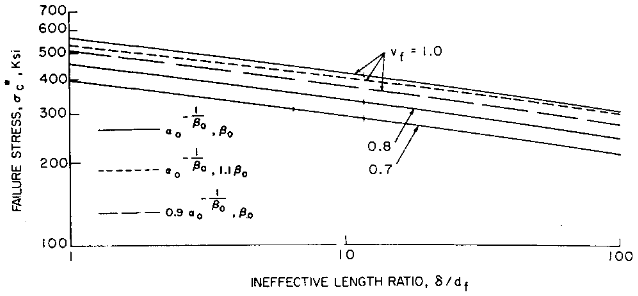

Fig. 6 Statistical mode of glass fiber-plastic composite failure stress.

where

$$\eta^2 = \left(\frac{G_b}{E_f}\right)\left(\frac{v_f^{1/2}}{1 - v_f^{1/2}}\right)\left(\frac{1}{r_f}\right)^2$$

and

G_b = shear modulus of binder
E_f = Young's modulus of fiber
v_f = volume fraction of fibers ($v_f = 1 - v_b$)
x = distance from fiber end
σ_{f0} = extensional stress in the fiber at a large distance from the fiber end

Similarly, the variation of extensional stress in the fiber is found to be

$$\sigma_f/\sigma_{f0} = 1 + \sinh\eta x - \cosh\eta x \tag{9}$$

It can be seen that the shear stresses are quite localized and decay rapidly as the fiber stresses approach their undisturbed value σ_{f0}. The fiber ineffective length can be quantitatively defined by specifying some fraction ϕ of the undisturbed stress value below which the fibers shall be considered ineffective. The fiber ineffective length δ can then be shown to be (when normalized with respect to the fiber diameter d_f)

$$\frac{\delta}{d_f} = \frac{1}{2}\left[\left(\frac{1 - v_f^{1/2}}{v_f^{1/2}}\right)\left(\frac{E_f}{G_b}\right)\right]^{1/2} \times \cosh^{-1}\left[\frac{1 + (1 - \phi)^2}{2(1 - \phi)}\right] \tag{10}$$

For illustrative purposes, a value of $\phi = 0.9$ is considered, and δ is evaluated for this stress ratio value. Thus, effective

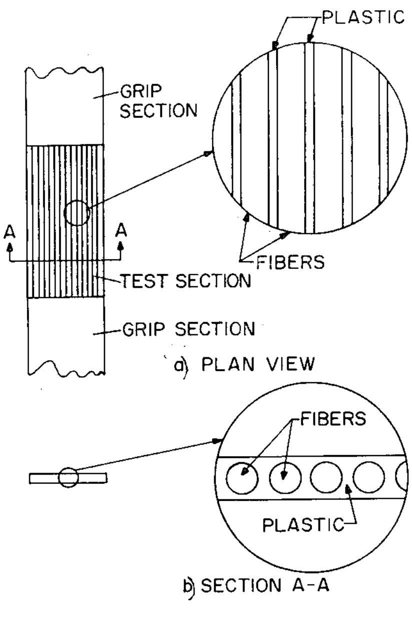

Fig. 7 Tensile failure test specimen.

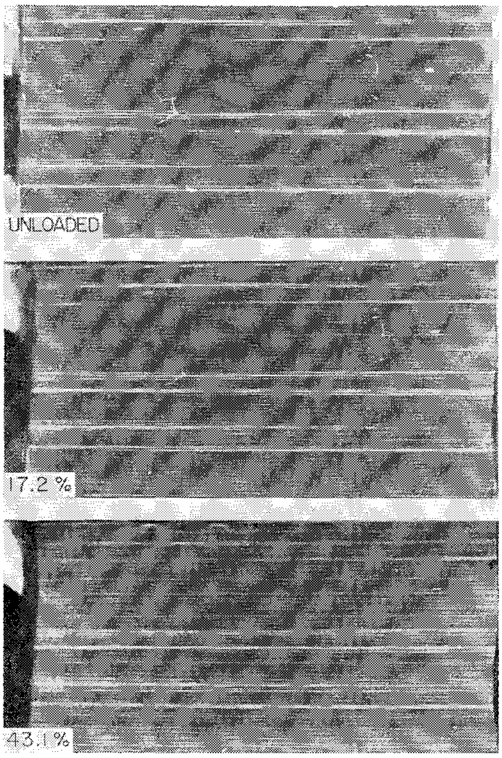

Fig. 8a

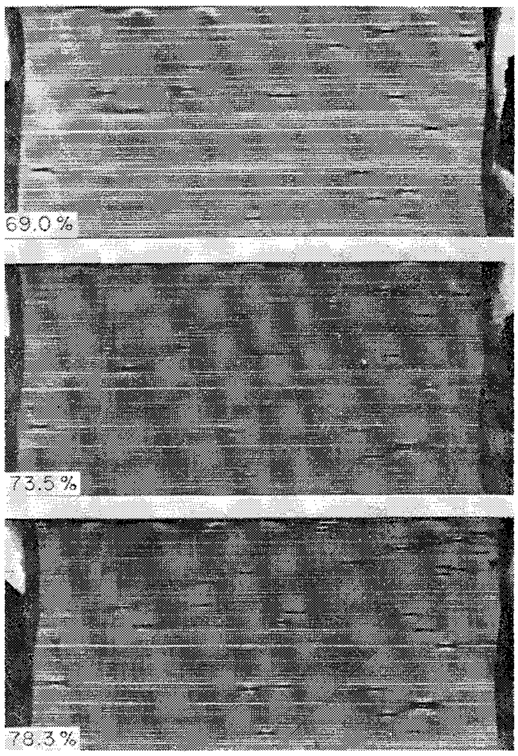

Fig. 8c

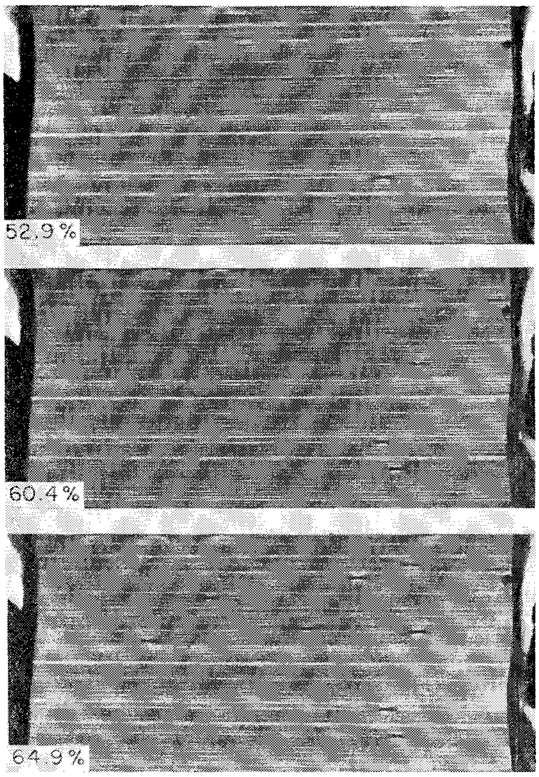

Fig. 8b

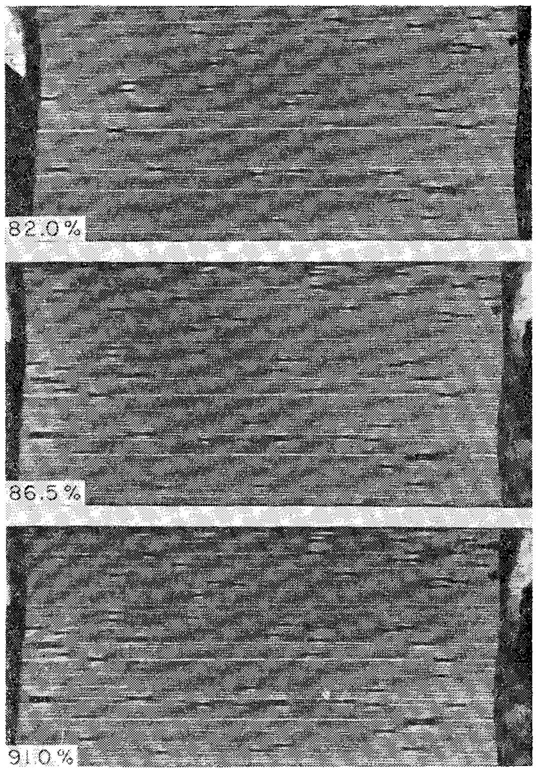

Fig. 8d

Fig. 8 Typical sequence of photographs of tensile failure specimen showing distribution of fiber breaks prior to failure (specimen A-7). Figure 8e is on the next page.

length is that portion of the fiber in which the average axial stress is greater than 90% of the stress that would exist for infinite fibers. Figure 3 shows the variation of ineffective length with constituent moduli for various fiber concentrations. The maximum stresses given by the analysis are shown in Fig. 4. It is clear that for many composites the matrix shear stresses will exceed the elastic limit of the material.

As a first approximation to the inelastic problem, the binder may be considered to have an elastic-plastic stress-strain curve. This combination of linear elasticity to a given yield stress and then to a constant stress for all larger strains

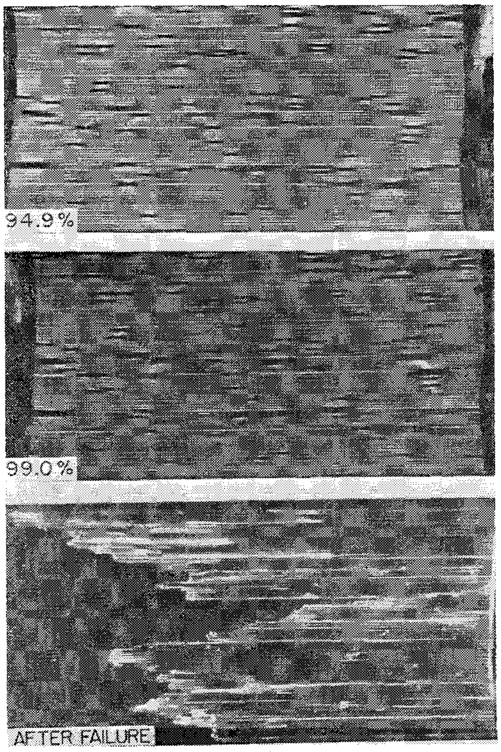

Fig. 8e (see preceding page for caption)

is a gross idealization of an actual epoxy stress-strain curve, but it does enable an estimate of the nature of inelastic effects. The resulting effects are that the maximum shear stresses are, of course, lower, whereas ineffective fiber lengths have increased. The stress decrease is beneficial, but the ineffective length increase can result in a reduced composite tensile strength as will be seen.

Fiber Link Strength Distribution

The statistical distribution of link strength is obtained from the fiber strength distributions. Consider links characterized by the distribution function $f(\sigma)$ and the associated cumulative distribution $F(\sigma)$ where

$$F(\sigma) = \int_0^\sigma f(\sigma)d\sigma \qquad (11)$$

For n such links forming a chain that fails when the weakest link fails, the distribution function $g(\sigma)$ for the chain is defined by

$$g(\sigma) = nf(\sigma)[1 - F(\sigma)]^{n-1} \qquad (12)$$

From this, the cumulative distribution function $G(\sigma)$ for the fibers is obtained:

$$G(\sigma) = \int_0^\sigma g(\sigma)d\sigma \qquad (13)$$

Therefore,

$$G(\sigma) = 1 - [1 - F(\sigma)]^n \qquad (14)$$

The solution of the inverse problem is desired. That is, given the fiber data $g(\sigma)$ and $G(\sigma)$, define the link data for a link length δ. From Eq. (14),

$$F(\sigma) = 1 - [1 - G(\sigma)]^{1/n} \qquad (15)$$

and thus from (11) and (15)

$$f(\sigma) = [g(\sigma)/n][1 - G(\sigma)]^{(1/n)-1} \qquad (16)$$

For a given set of experimental data $g(\sigma)$ for the tensile strength of a fiber of length L ($L = \eta\delta$), Eq. (16) can be used to characterize the fiber links.

Typical Application

As an example, consider fibers characterized by a strength distribution of the Weibull[10] type:

$$g(\sigma) = L\alpha\beta\sigma^{\beta-1}\exp(-L\alpha\sigma^\beta) \qquad (17)$$

This form has been shown to characterize the experimental length L to strength σ relationship of fibers. Using Eq. (17) in (13) and (16) yields

$$f(\sigma) = \alpha\delta\beta\sigma^{\beta-1}\exp(-\alpha\delta\sigma^\beta) \qquad (18)$$

where $L = n\delta$.

The constants α and β can be evaluated by using experimental strength-length data. To do this, consider the mean fiber strength $\bar{\sigma}_f$ for a given length, which is defined by

$$\bar{\sigma}_f = \int_0^\infty \sigma g(\sigma)d\sigma \qquad (19)$$

Substituting Eq. (17) into (19) and integrating yields

$$\bar{\sigma}_f = (L\alpha)^{-1/\beta}\Gamma[1 + (1/\beta)] \qquad (20)$$

A logarithmic plot of the available data[6] for $\bar{\sigma}_f$ as a function of L will define the constants. Such a plot is presented in Fig. 5. The linearity of the data support the choice of the distribution function given by Eq. (17). It should be noted that a good fit to the experimental data in the form shown in Fig. 5 can also be obtained with distribution functions other than that of Eq. (17). For the present illustrative purposes, however, the Weibull distribution will be used. The constants are found to be

$$\alpha = 7.74 \times 10^{-20} \qquad \beta = 7.70 \qquad (21)$$

The constant β is an inverse measure of the dispersion of material strength. Values of β between 2 and 4 correspond to brittle ceramics, whereas a value of 20 is appropriate for a ductile metal.[7] The constant α, as seen from Eq. (20), defines a characteristic stress level $\alpha^{-1/\beta}$. For this distribution, $\alpha^{-1/\beta}$ is 305 ksi.

With the fiber link length and strength distribution function defined, Eq. (7) can be used to determine the composite strength. The most probable failure stress σ_c^* is obtained by setting

$$d\lambda/d\sigma_c = 0 \qquad (22)$$

Substituting (7) into (22) yields[11]

$$\sigma_c^* = \sigma_B - \psi_B(2\log n)^{1/2} + \psi_B\frac{\log\log n + \log 4\pi}{2(2\log n)^{1/2}} \qquad (23)$$

For composite dimensions large compared to fiber cross-section dimensions, $N \gg 1$. Therefore, $\psi_B \to 0$, and the statistical mode of the composite strength σ_c^* is found to be

$$\sigma_c^* = (\alpha\delta\beta e)^{-1/\beta} \qquad (24)$$

where α and β are the constants defining the link strength and are determined by experimental tests of fiber strength vs length as described previously. δ is the ineffective length defined by a fiber shear stress analysis, and e is the base of natural logarithms.

The results of the preceding sections are used in Eq. (24) to compute composite strength. The predicted composite failure stress is plotted in Fig. 6 for the range of ineffective lengths of 1 to 100 fiber diameters. The range 1 to 10 generally corresponds to the elastic predictions (see Fig. 3) and the range 10 to 100 to the inelastic predictions. Also shown in Fig. 6 are the effects of variations in fiber characteristics. Curves are presented to show the effect of an increase in the dispersion, as measured by a 10% change in β, and of a decrease in the reference strength, as measured by a 10% change in $\alpha^{-1/\beta}$. Note that, as the dispersion in fiber strength becomes small, the fibers are characterized

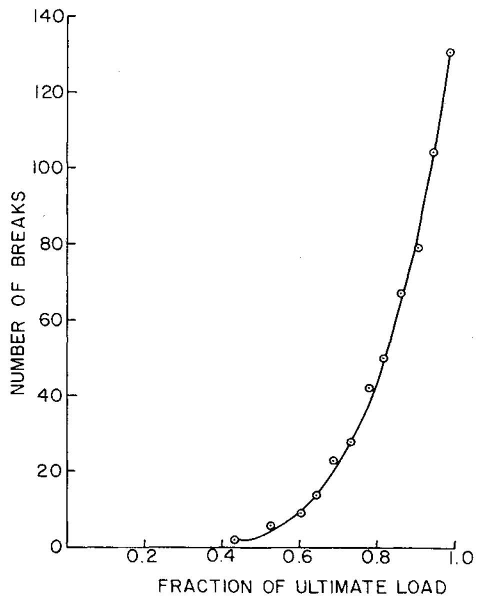

Fig. 9 Number of fiber breaks in specimen A-7 as a function of applied load.

by a single strength value rather than by a distribution function, and the composite will fail when the fibers reach this stress value. The failure stresses shown are the average fiber stresses, and hence a composite failure stress is obtained by multiplying the value for $v_f = 1.0$ (which case has no physical equivalent) by the actual fiber volume fraction.

Experimental Treatment

The experimental study of the mode of failure of fiber reinforced composites under a tensile load, which was undertaken to qualify the analytical model, utilized specimens consisting of a single layer of parallel glass fibers imbedded in epoxy. The specimen, as shown in Fig. 7, has a test section that is $\frac{1}{2} \times 1$ in. in size and 0.06 in. thick and contains 90–100 parallel glass fibers of 0.005-in. diam. The specimen is loaded in tension and observed microscopically during the test. The design of the specimen was directed toward making this observation possible, so that the nature of failure of fibrous composites could be determined. In particular, the validity of the failure mode used in the preceding analysis was to be tested.

Still and motion pictures have been taken of the tests. A sequence from a typical set of photographic data is shown in Fig. 8. The load was applied parallel to the fibers. Note again that the fiber diameter is on the order of five times the minimum distance between fibers. The first frame shows the specimen at zero load. Polarized transmitted light has been used, and at zero load the fibers are dark and the binder between fibers appears light. As the load is increased, the fibers appear lighter, although this is not adequately reflected in the photos since the lens aperture was changed during the sequence. The difference between the unloaded specimen photo and the next to the last photo in the sequence is four f-stops, or a factor of 16 on the exposure.

At less than 50% of the ultimate load, individual fiber fractures are observed. Since the fractured fiber in the vicinity of the fracture is unstressed, the color returns to the original dark color. Thus, a break appears as a short dark rectangular area with a thin white line across the center. The length of this dark area is the ineffective length of the fiber. As the load increases, the fibers fracture at random locations. Although there are stress concentrations in the vicinity of the breaks, the variation in fiber strength generally more than offsets the effect of such concentrations. Hence, the breaks occur randomly rather than cumulatively at the site of the initial break. The stress concentrations cause a relative brightening at the highly stressed points of the fibers, and this effect appears on the latter photos in the sequence of Fig. 8. Also, there are examples of breaks that were produced as a result of the stress concentrations. The specimen is shown in the last frame after fracture. The data for the specimen of Fig. 8 are plotted in Fig. 9. These results are typical of the scatter of data points around the best fit curve. Such curves for the series of test specimens described in Table 1 are shown in Fig. 10.

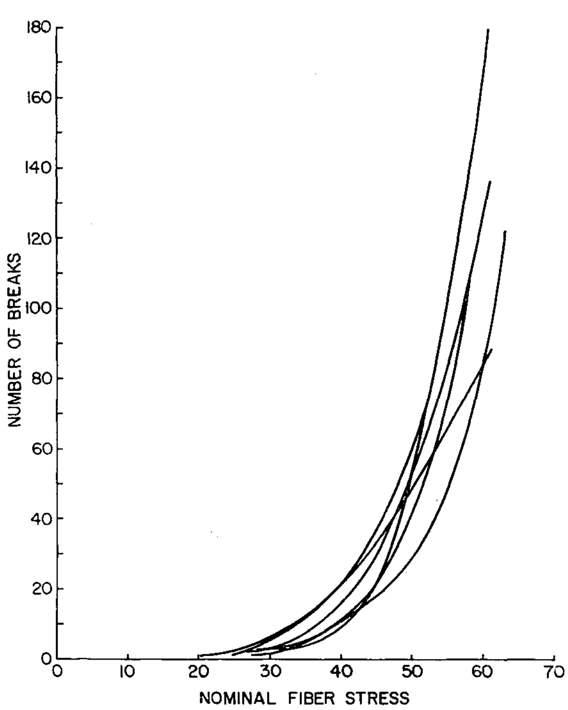

Fig. 10 Number of fiber breaks as a function of nominal fiber stress for series A specimens.

Discussion and Conclusions

A statistical analysis of the tensile failure of fibrous composites has been performed. The analysis attempts to simulate a failure mode based on distributed internal fractures prior to composite failure. Although an effort was made to approximate the most important parameters influencing failure, the model is not likely to provide accurate quantitative answers without further refinements. It is expected, however, that the nature of desirable improvements in constituent characteristics can be ascertained from the present model. The shortcomings of the model include failure to consider fracture involving parts of more than one

Table 1 Tensile strength tests: series A

Spec. number	Gage length, in.	Width, in.	Thickness, in.	Number of fibers	Ultimate load, lb
1	1.10	0.499	0.0066	92	114
2	0.91	0.498	0.0060	93	84[a]
3	1.10	0.500	0.0067	93	111
4	0.90	0.502	0.0064	94	125
5	1.05	0.494	0.0061	92	116
6	1.02	0.500	0.0062	94	117
7	1.00	0.498	0.0062	93	116
8	1.07	0.503	0.0061	...	65[a]
9	1.03	0.490	0.0060	93	107

[a] Failure in grip section. Test data not used.

layer, variation of ineffective length with stress level (for the inelastic case), and stress concentrations in fibers adjacent to failure areas and the initial state of stress. Further, it will be necessary to obtain an accurate statistical characterization of any given fiber population so that the link strength characterization can be inferred properly. On the positive side, however, the model represents the constituents in the major functions of fibers carrying extensional stress and matrix carrying shear stresses; it includes the effect of fiber imperfections on fiber failure, and it accounts for the accumulation of internal cracks that combine to produce composite failure. This latter follows the concepts of Parratt,[3] who suggests the influence of flaws and ineffective lengths on failure. The failure mode in the present analysis, however, results from an accumulation of cracks rather than from the existence of fully ineffective fibers. In fact, the present results, which indicate typical fiber lengths at failure and which are an order of a magnitude larger than the ineffective length, perhaps explain the quantitative difference between the ineffective lengths obtained by Parratt[3] and those of Dow[5] and this paper.

The present analytical and experimental studies indicate that discontinuous fibers may exist in a composite of originally continuous fibers at stress levels well below the maximum load. The mechanical properties of the matrix material will then have a significant effect on the composite strength. This influence is measured by the efficiency with which the matrix transmits load around a fiber break. The experimental techniques developed to observe the internal failure process appear to give consistent results. The expanded use of such techniques appears to offer promise of increased understanding of the failure process.

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